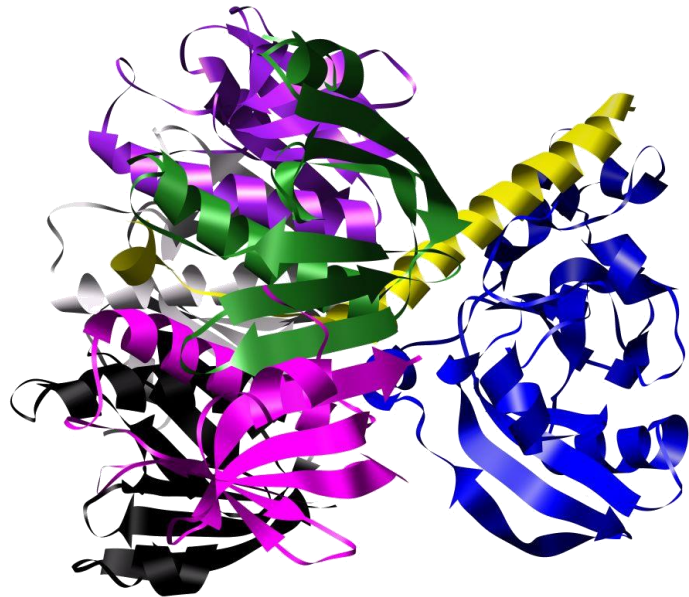


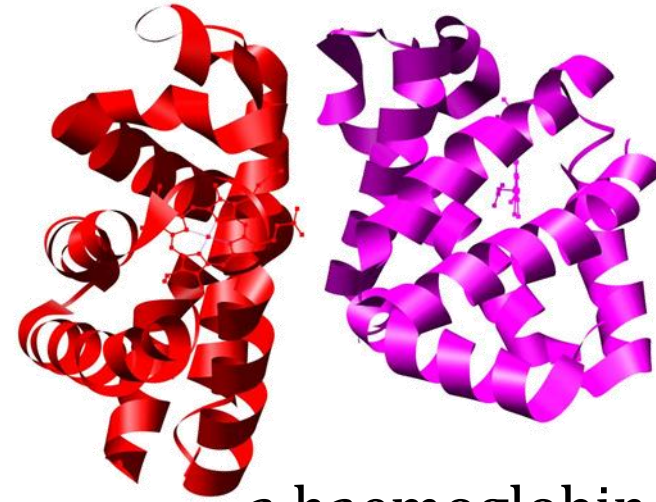
# Protein Domains

Two weeks for this topic

- many proteins have separate chains



enterotoxin  
(1lts) 7 chains



α haemoglobin  
(1h97)

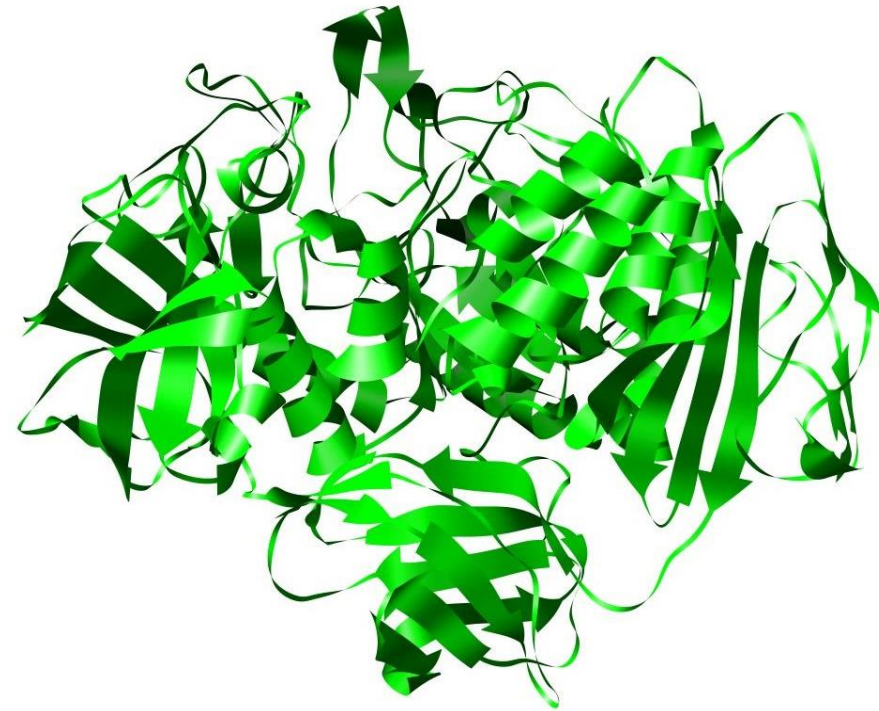
- what about units within one chain ?

# 4 domain protein

1cxl has 686 residues

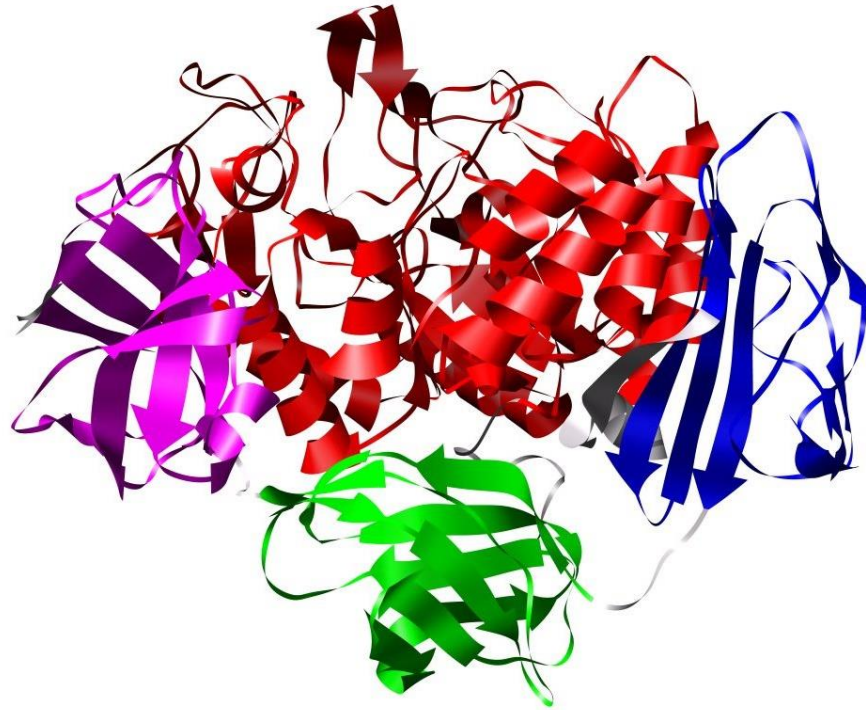
Cleaves carbohydrate bond

- one solid lump but...



# 4 domain protein

1.  $\alpha$ -amylase catalytic
2.  $\alpha$ -amylase C-terminal
3. immunoglobulin like domain
4. starch binding



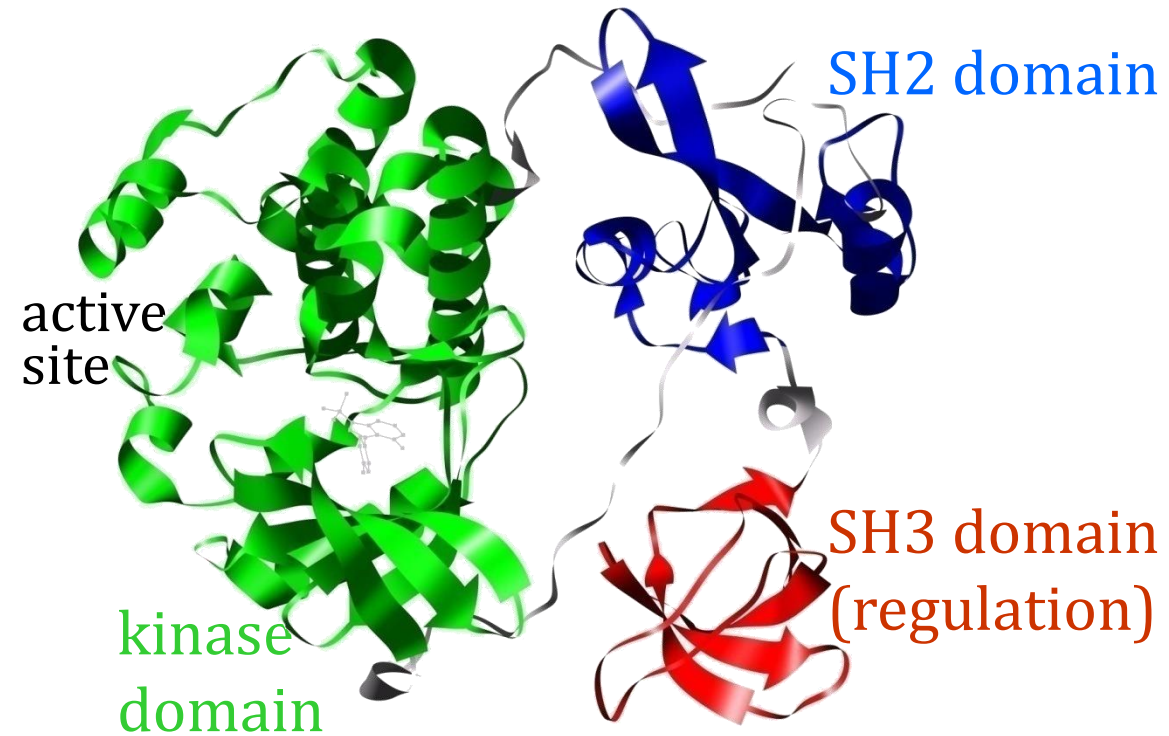
- even clearer example

# 3 domain protein

1qcf "src tyrosine kinase"

The domains really are common  
to other proteins

Number of domains is  
not absolutely defined



# Plan

- chemistry, examples
- methods to automatically recognise domains (examples)
- chemistry – how common are domains of different sizes, types, ...

# Earlier history

Term "domain" used before there were many structures

- Invented example: protein that
  - joins  $\text{ADP} + \text{P}_i \rightarrow \text{ATP}$
  - performs some oxidation
  - responds to some regulator
- take protein + protease (splits protein in a few places)
- cleave / break protein - get a few pieces (2, 3, 4..)
- purify pieces
- pieces found that
  - can bind ADP/ATP
  - bind sugars, some regulators

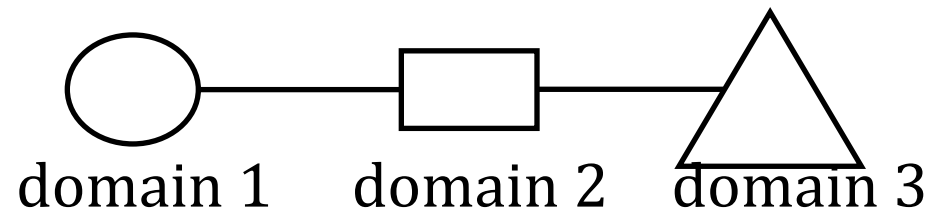
# Earlier history

Appeared that for some proteins

- different functions associated with different pieces
- refer to as "functional domains"

Belief / claim

- bigger proteins are made from units, combined over evolutionary time scales



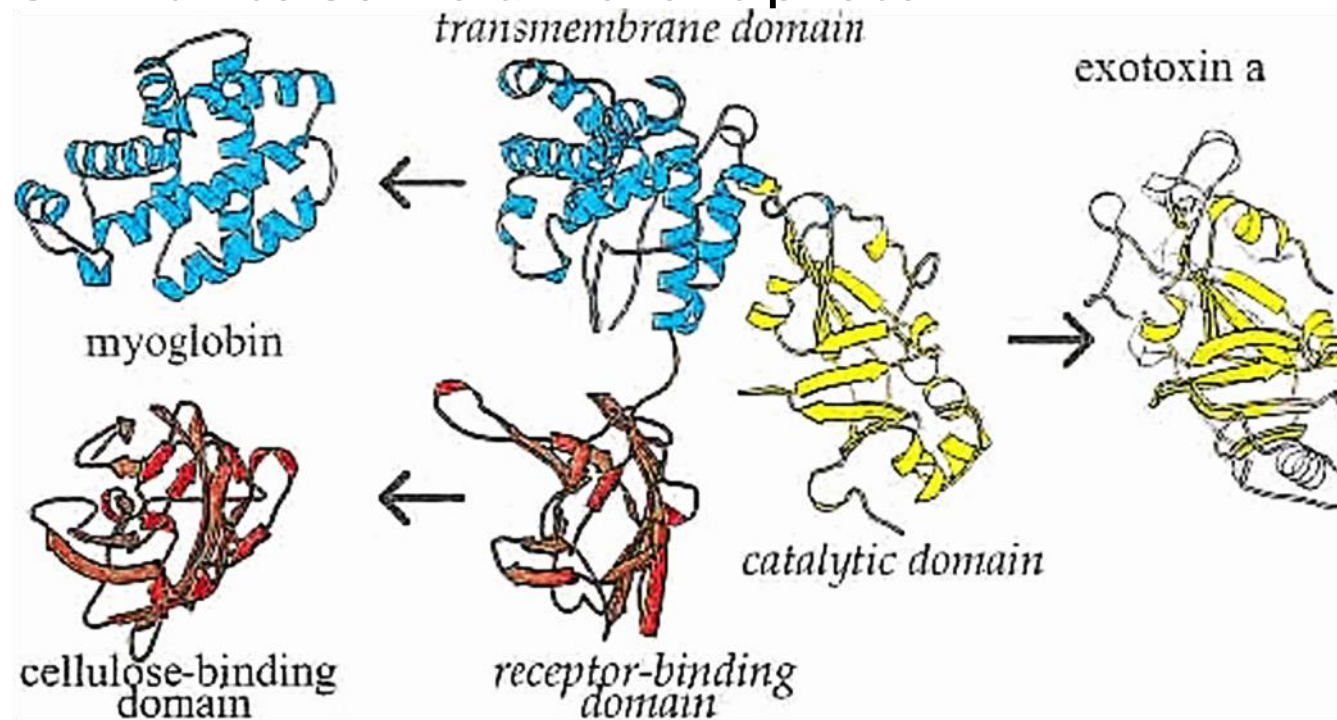
- an example...



# modular protein

Diphtheria toxin (1ddt) middle of picture

- 3 domains
  - each similar to some different protein

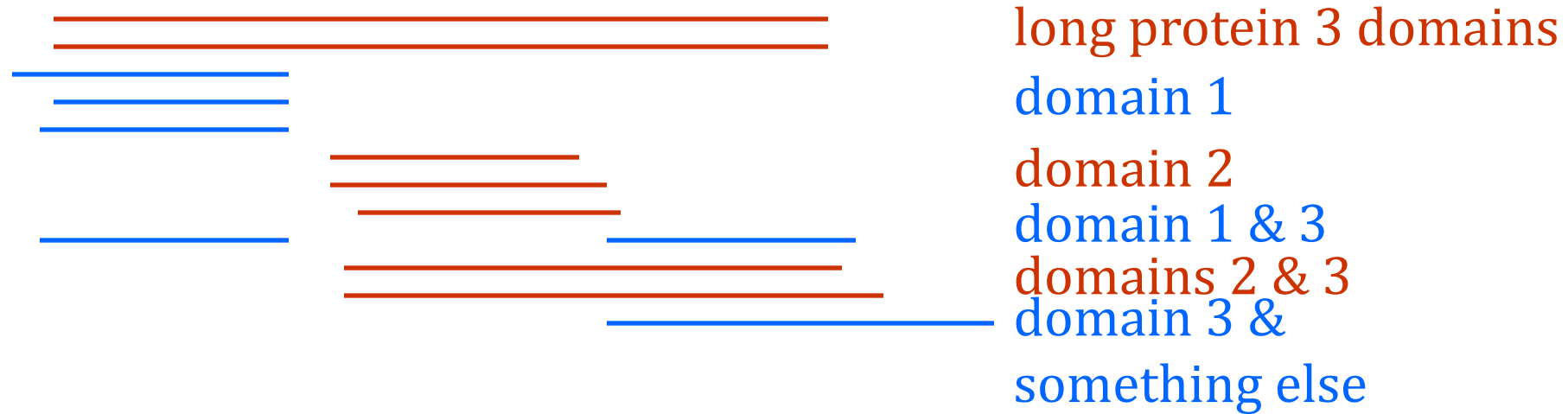


- appears as if modules are mixed together
- should be visible at sequence level...



# Sequence level domains

Align a group of sequences



- appears to have 3 or 4 domains
- no reference to structures or function

# Domain definitions summary

	structure	sequence	biochemistry
functional	not necessary	not necessary	yes
sequence-based	not necessary	yes	no
structure	yes	usually known	no

How important ?

- $> \frac{2}{3}$  proteins have 2 or more domains
- part of definition
  - a piece of a protein which can fold and is stable

Now

- methods based on structure

# Finding Domains

A definition leads to methods

- domain is a compact unit

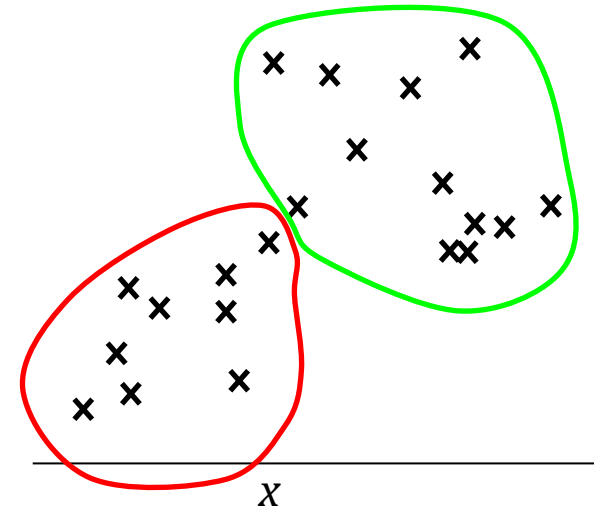
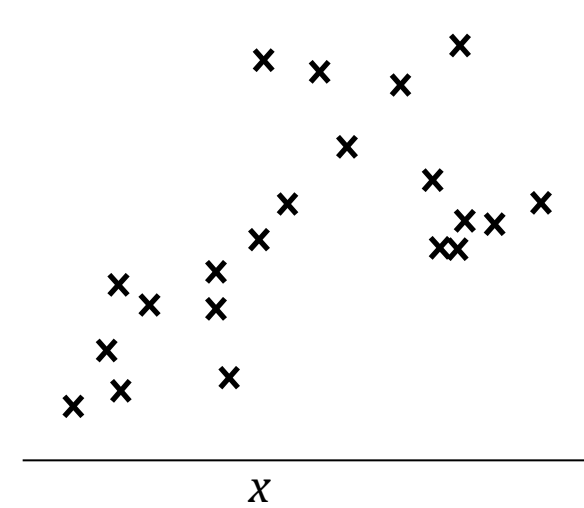
Objective way to look for dense units ?

- cluster analysis

Philosophy in cluster analysis

- look for dense groupings

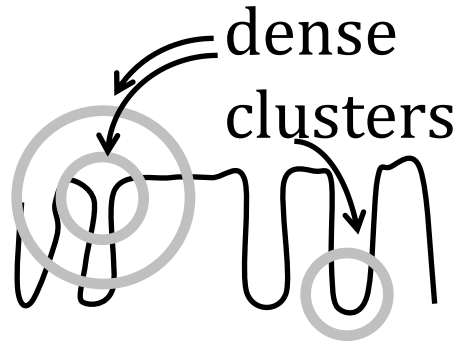
Leads to dendrogram



# Clustering

## Approach

- need a (dis)similarity matrix between every object
- here: distance between  $C^\alpha$  atoms



residue	1	2	...	N
1	0	...	...	...
2		0	...	
...			$\ddots$	
N				0

- does this work ?

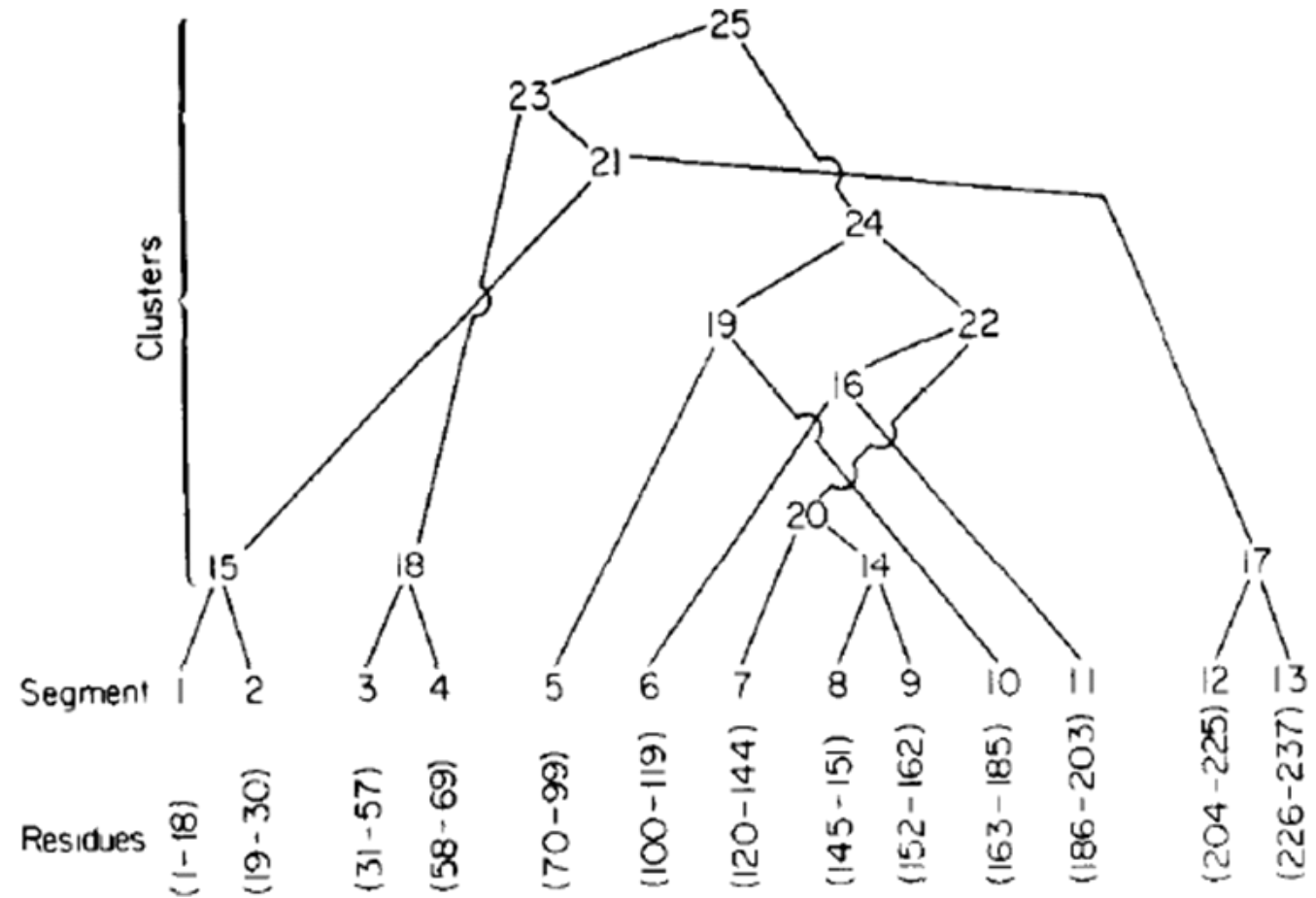
# Clustering

Clustering applied to concanavalin A

- bottom - small compact pieces
- higher – compact units
- looks like natural  
3 domains

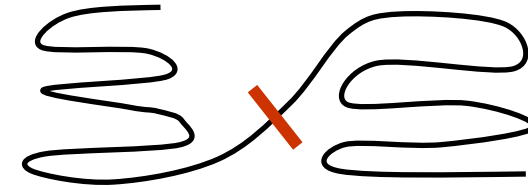
Number of domains is not  
absolutely determined

Very very very old method



# Cuts / Surface area / volume

- Simple idea - cut chain in two pieces
- density of part 1 / versus part 2
- cut so as to maximise density



## Problems - one cut is not enough



A method should be able to split with 1, 2, 3, ... cuts

- For 3 cuts with  $N_{res}$  positions:  $N_{res} \times N_{res} \times N_{res}$

really  $(N_{res})^{N_{cut}}$

# Problems - density

I want to maximise density

- density of protein ?
  - number of residues in a volume ?
  - volume ? not sphere

Contacts are easier than density

- within a domain there are many contacts
- between domains - few contacts
- an approximation



# Counting contacts

Do I have many contacts compared to the number of atoms ?

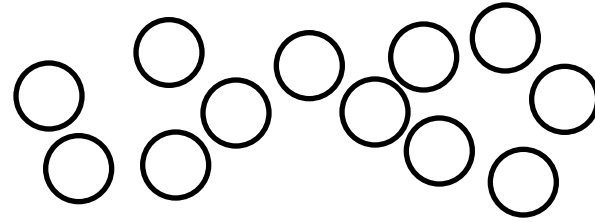
- calculate distance between each  $C_i^\alpha C_j^\alpha$  atoms =  $d_{ij}$
- if  $d_{ij} < 4 \text{ \AA}$ , set  $p_{ij}=1$  else  $p_{ij} = 0$
- for a given set of  $N_{res}$  atoms (not whole protein)

$$\frac{\sum_{i=1}^{N_{res}} \left( \sum_{j>i}^{N_{res}} p_{ij} \right)}{N_{res}}$$

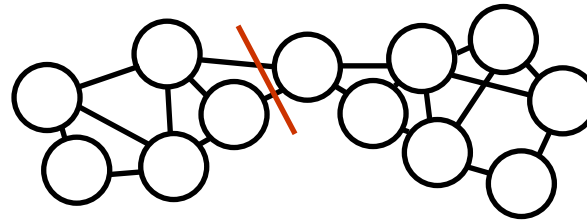
- not accurate, but easy to calculate

# Cutting / contacts

A protein



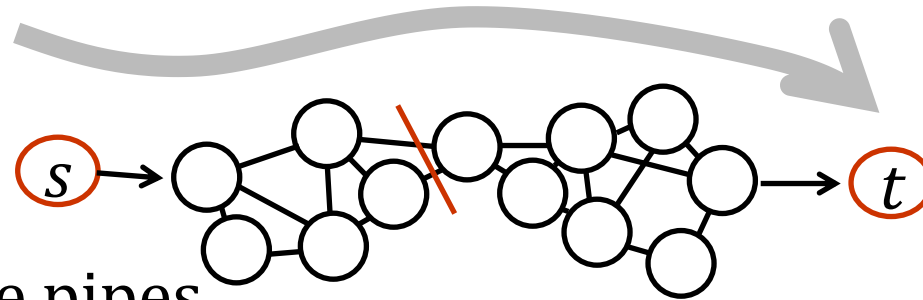
Find close contacts



How can one find the best place(s) to cut ?

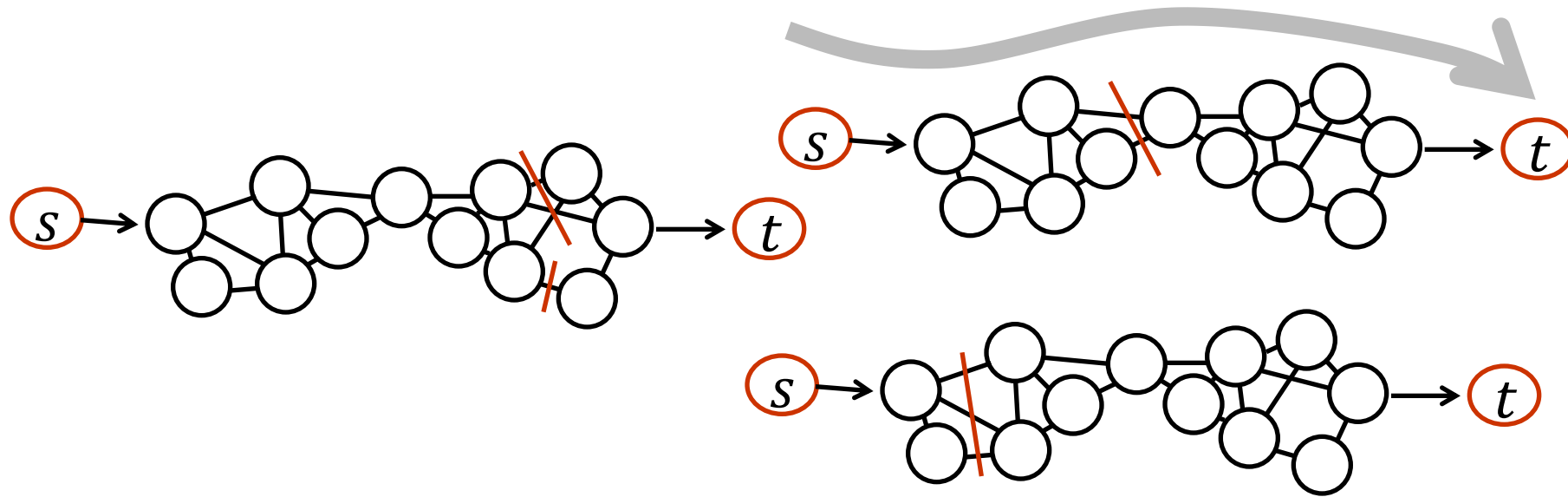
Feed water into  $s$  (to  $t$ )

- find the most blocked restrictive pipes
- not one, but all that are restrictive



# Cutting / contacts

- Flow problem
- Many ways to cut the flow from  $s$  to  $t$

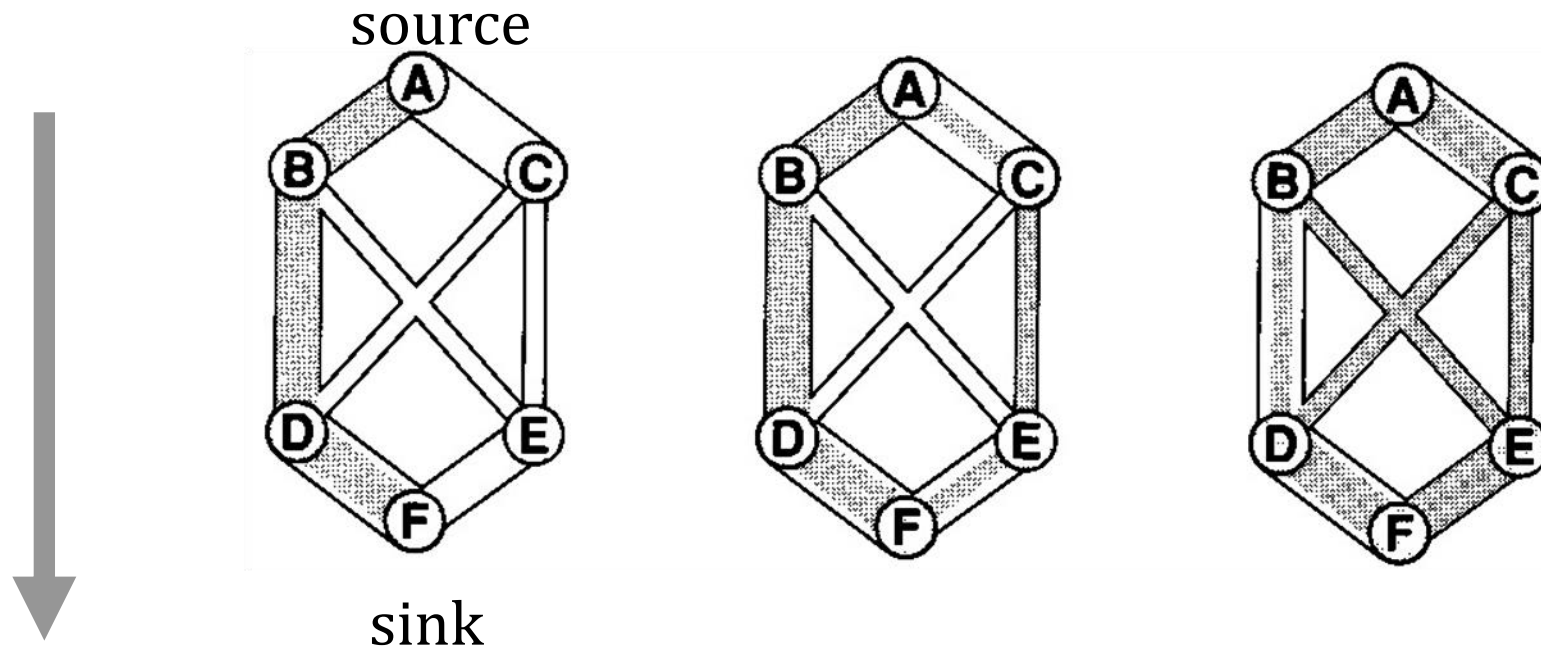


- of all these " $st$  cuts" find the one with smallest capacity (flow)
- more interesting - make the pipes different flow capacity
  - how are the residues really touching ?  $C^\alpha C^\alpha$  or  $C^\alpha$  sidechain

# Cutting / contacts

Two steps

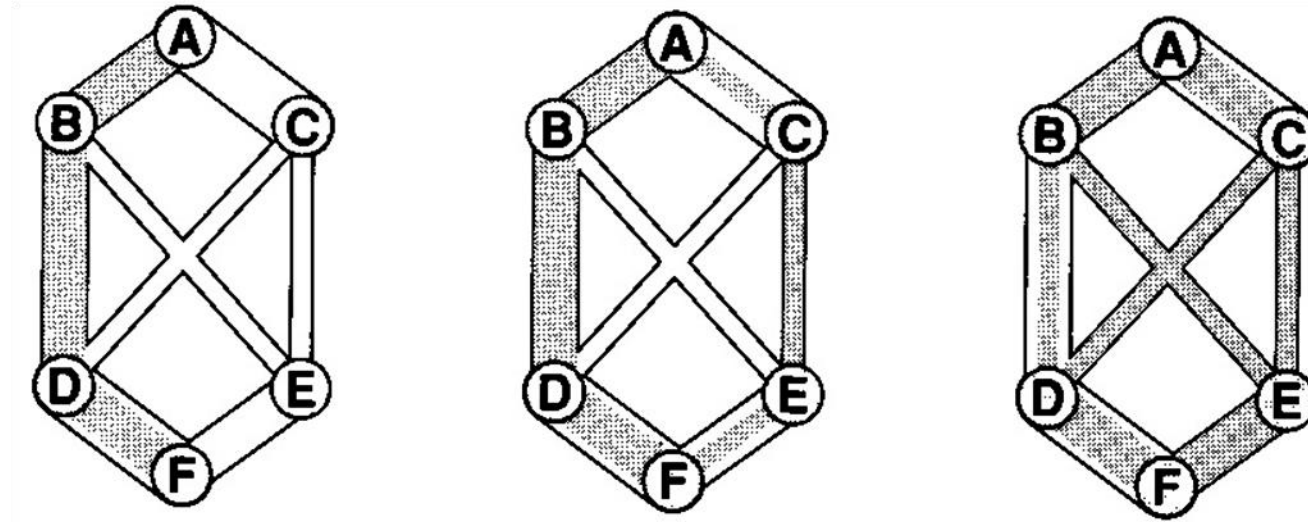
1. find the maximum flow from  $s$  to  $t$
2. cut  $s$  from  $t$  at the few most filled pipes



# Maximum Flow

rule

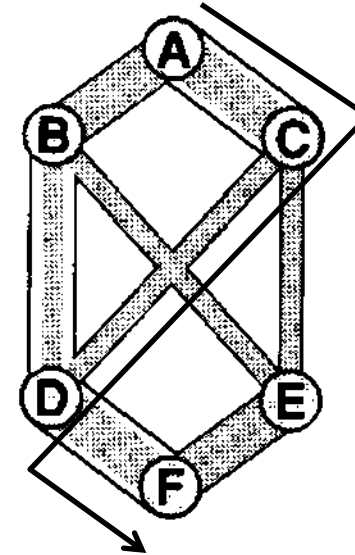
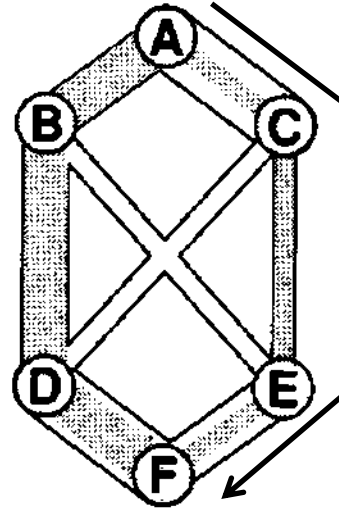
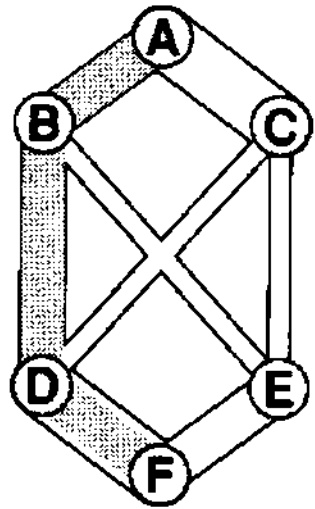
- if every path from source to sink has one full edge
  - flow is maximum



- keep trying every possible path, look to see if there is unused capacity
- we can go backwards

# Maximum Flow

- A B D F                      note DF is not quite full
- add some A C E F              AC, EF are not full
- look at C, switch some capacity to CD (DF)

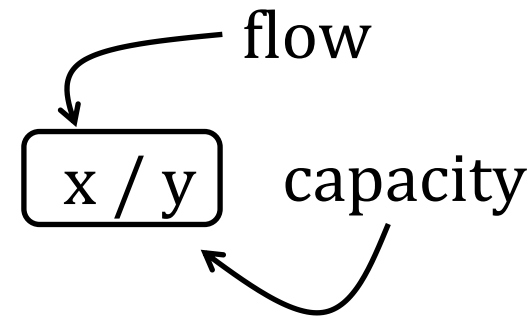
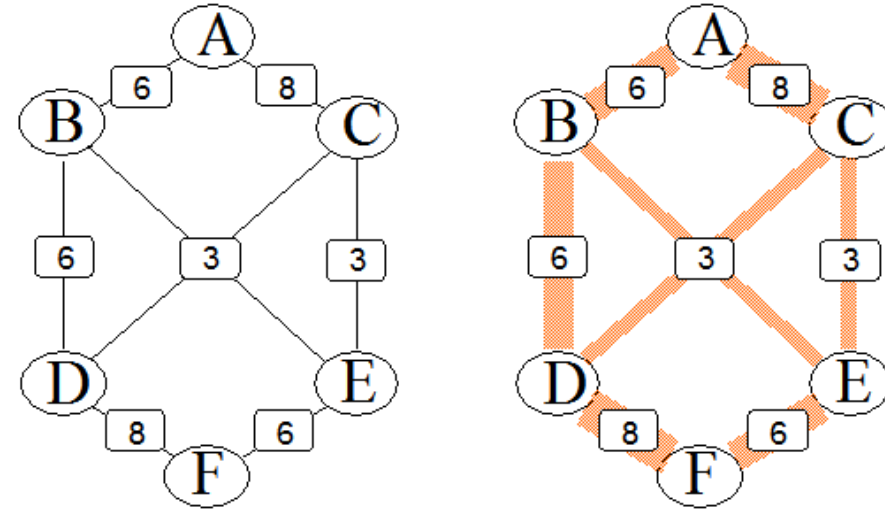
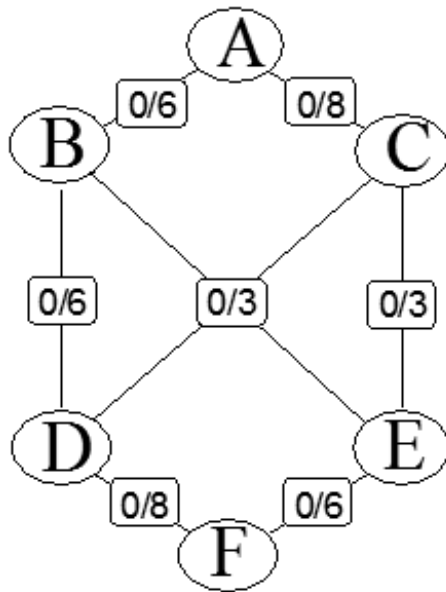


- with some numbers on the edges

# Maximum Flow

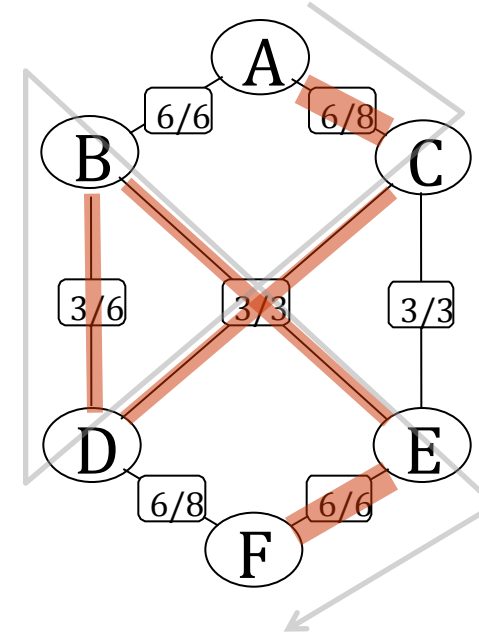
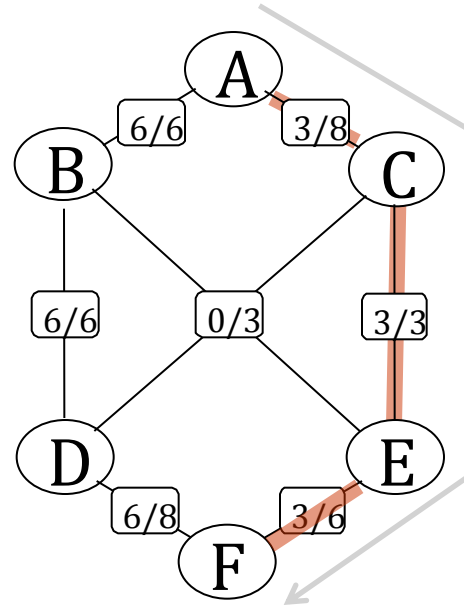
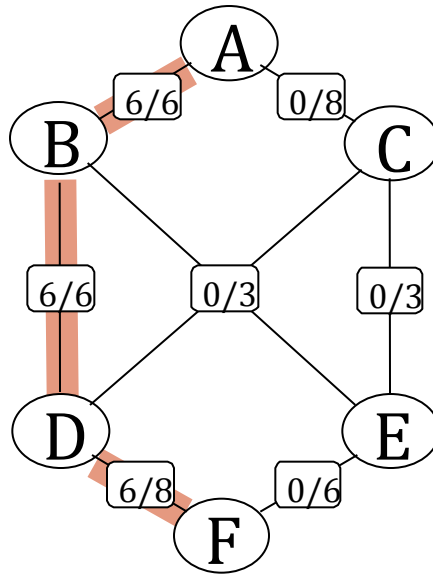
Define an example system

- flow into A
- out of F
- capacities at each edge vary





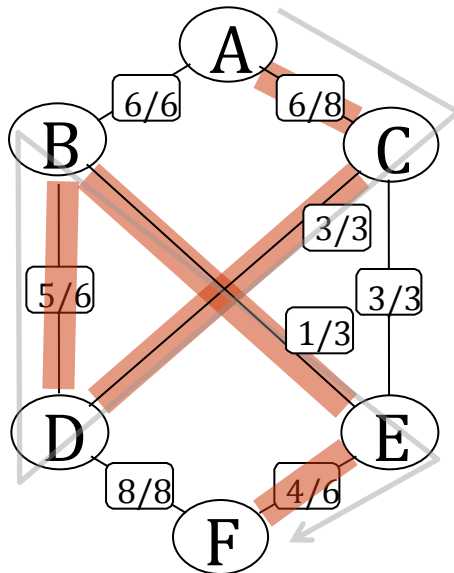
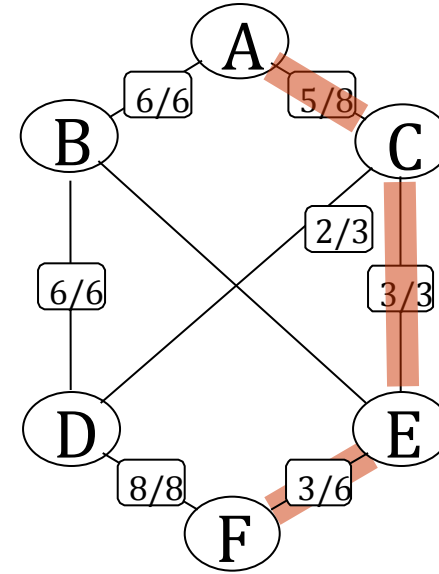
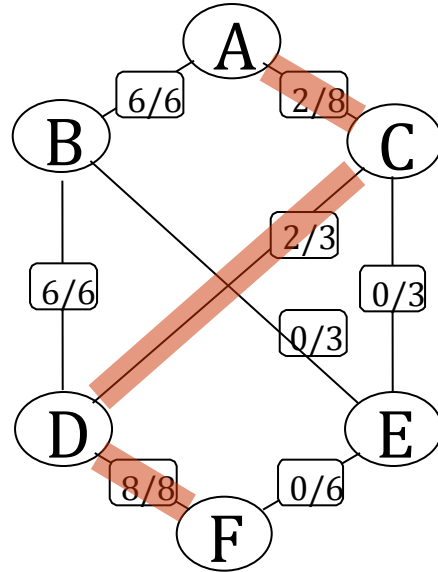
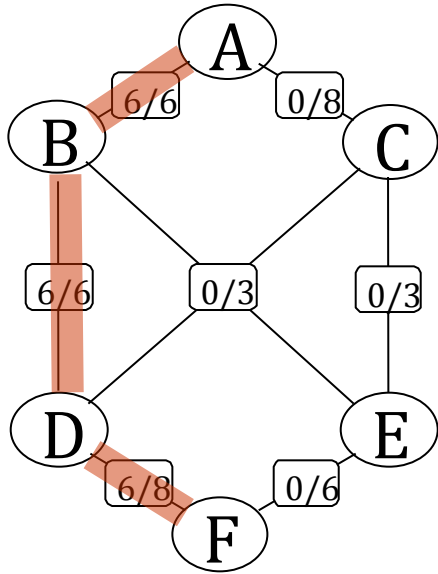
# Maximum Flow



Find a new path (possibly with backwards flow)

- what is the smallest unused capacity on the path ?  $\Delta f$   
 $> 0$  ? send flow  $\Delta f$  in this path

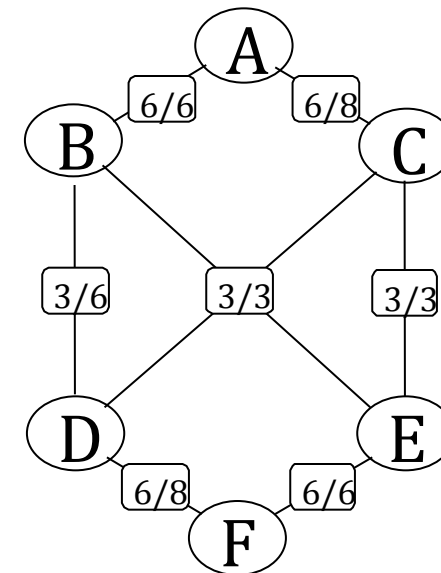
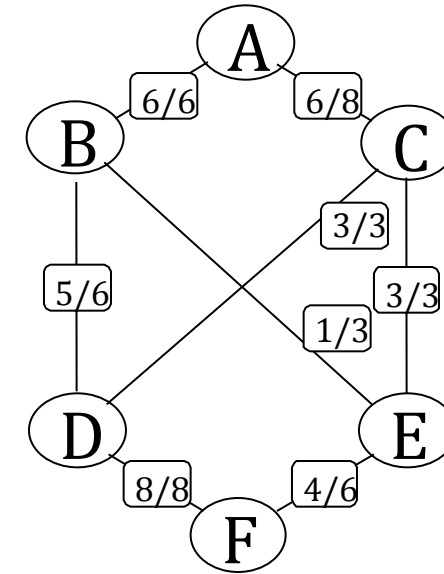
# Alternative



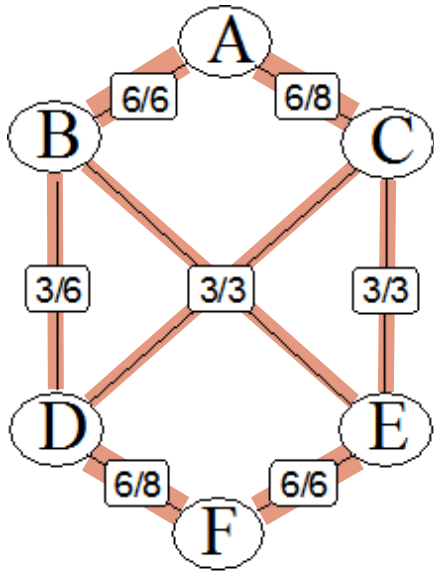
- also ends with flow of 12
- look why this this is definitely time to stop ...

# Alternative

- are there any paths with unfilled pipes ?
- start at A
  - to the left is filled
  - try  $A \rightarrow C$ 
    - both routes out of C are filled
- more solutions ?
- definitely different ways to find solutions
  - different order of visiting paths



# Maximum Flow



- path = any route from A to F
- is there any path where all edges have extra capacity ?
  - finished - flow is maximum
- algorithm (not optimal)  

```
while (flow not maximum / path found)
    add flow to path
```

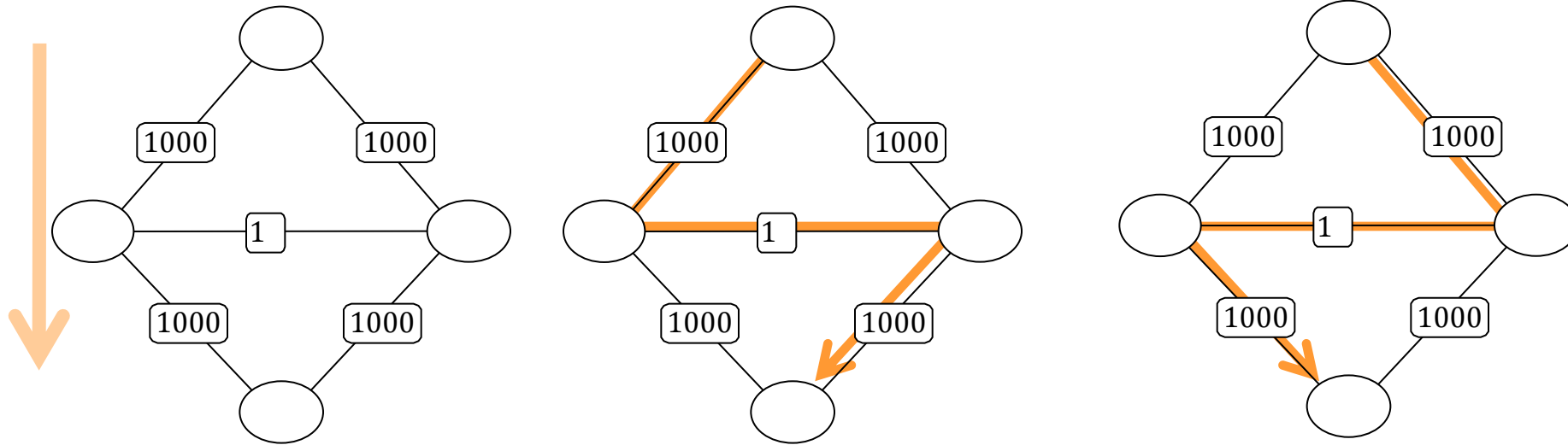
Our definition - finished when

- every path from source to sink has at least one edge (pipe) which is full

Is this efficient ?

# Efficiency

Worst possible selection of path order would require 2000 iterations



First part of procedure finished

- flow is maximum
- next
  - where to cut graph

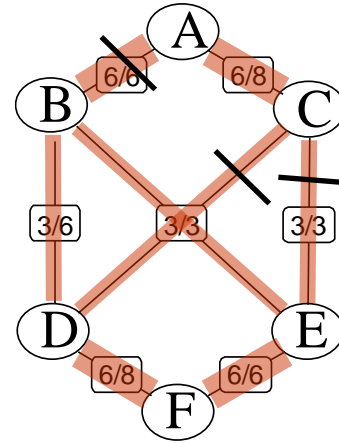
# Cutting graph

Find ways to cut network, max flow = 12

- AB, AC capacity = 14
- BD, BE, AC capacity = 17
  - both bigger than flow (12)

Better

- for each path
  - find first full pipe - cut
  - AB, CD, CE capacity = 12
  - = max flow
  - best cut



# Cutting graph

If the capacity across our set of cuts = maximum flow

- it is a "minimum cut"
- smallest connection between two parts of graph
- graph / network / protein is broken into two parts / domains

Useful yet ?

- no mention of finding source  $s$  and sink  $t$
- details - efficiency not mentioned



# Network flow and proteins

## Source

- find a surface residue
- connect an  $s$
- connect to nearby surface residues

## Sink

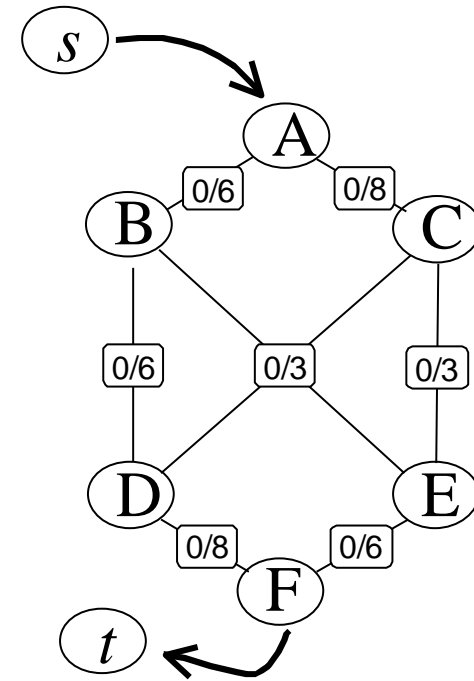
- find a surface residue far away, connect to  $t$

*Ad hoc* ? arbitrary ? optimal ?

- maybe not critical

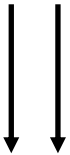
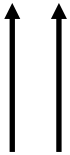
Multiple domains ?

**while (domains not too small)**  
**keep trying to split**

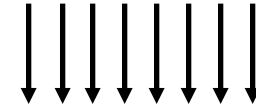


# Splitting - near neighbours / Ising spins

Background story - Ising spin model

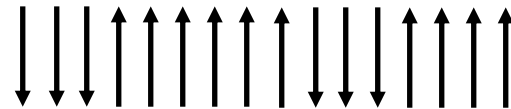
- energy of spin  $i$  depends on  $i, i + 1$
- energy can be good  or 

or bad 



For lots of spins

- islands of same spin



- can be generalised to 2D, 3D

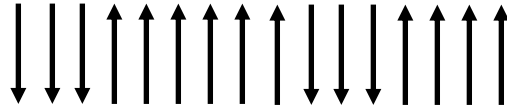
Finding low energies ? Simplest method

- try to flip a random spin
  - accept flip if energy improves
  - sometimes accept if energy goes up (probabilistic)

# Splitting - near neighbours / Ising spins

Slightly better method

while (energy still high)  
    for each spin  
        change to be same as average of  $n$  neighbours

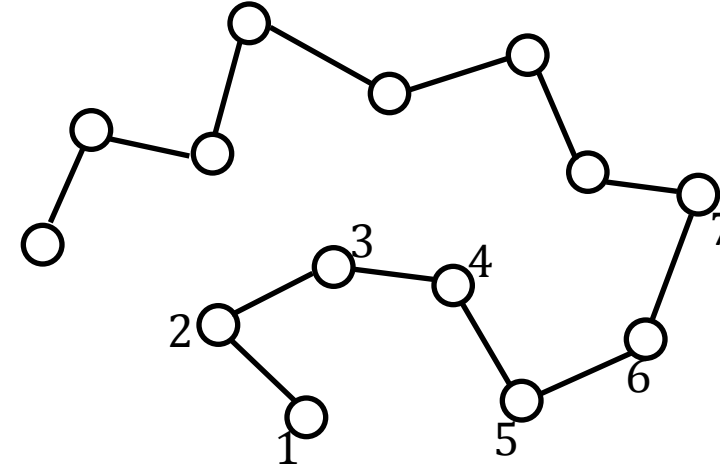
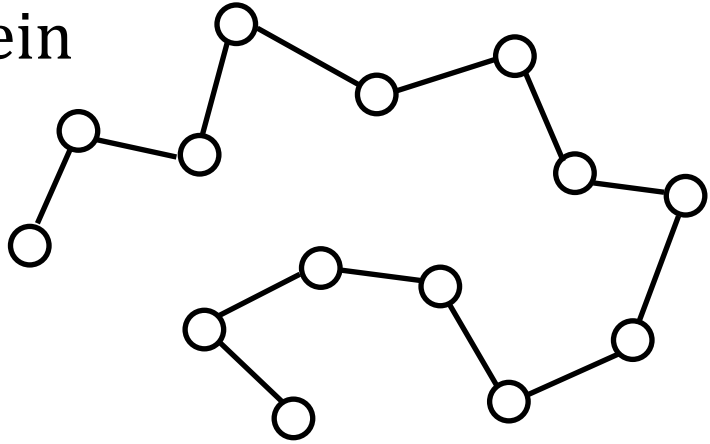


Protein version

- for any known structure
  - easy to make list of neighbours of each residue
- residues close in space should be in similar domains

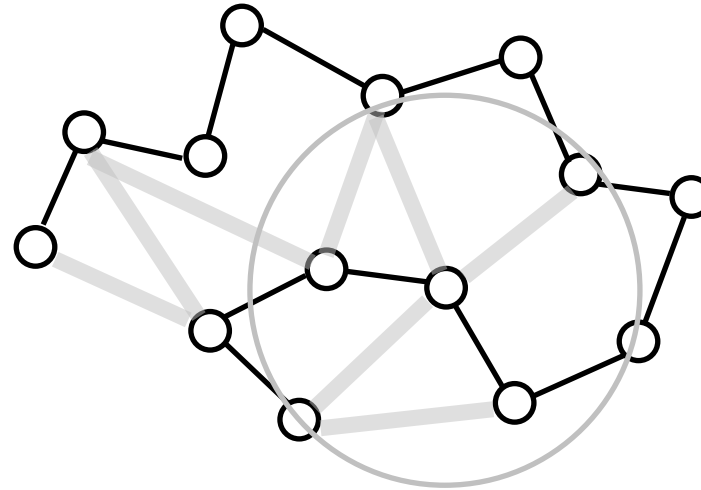
# Splitting - near neighbours / Ising spins

protein



label all points  
with a number

make a list of neighbours for  
each residue



# Splitting - near neighbours / Ising spins

label of a residue is  $m_i$   
while (labels changing)  
for each residue  $j$

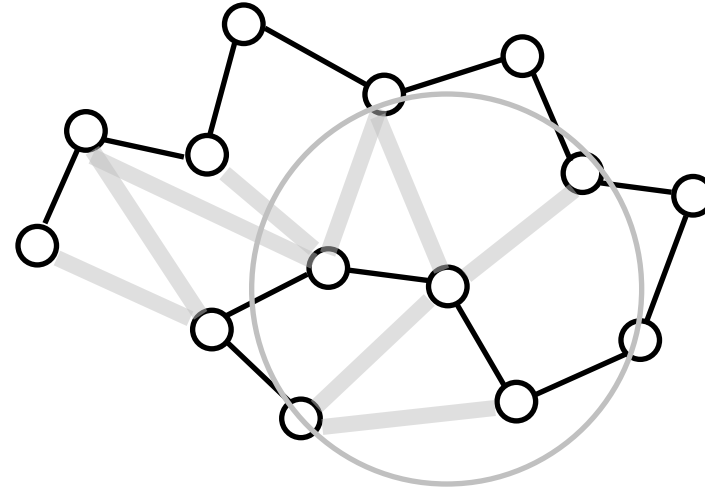
$$m_{av} = \frac{\sum_{i \in \text{neighbours}} m_i}{n_{\text{neighbour}}}$$

if ( $m_{av} > m_j$ )

$$m_j(\text{new}) = m_j(\text{old}) + 1$$

else if ( $m_{av} < m_j$ )

$$m_j(\text{new}) = m_j(\text{old}) - 1$$



step	residue number								
0	1	2	3	4	5	6	7	8	...
1	2	3	3	4	6	7	8	8	...
2	2	2	4	3	5	8	9	9	
...									

# Splitting - near neighbours / Ising spins

Properties of Taylor / Ising spin-inspired method

Optimism

- will converge and become stable

Requires threshold - what is a neighbour

- can use sophisticated averaging - distance dependent
- may converge to 2, 3, ... domains

# Methods so far

1. simple - look for single cut points and maximise density
  2. Crippen / hierarchical clustering
  3. Network flow
  4. Ising spin / Taylor
- All methods have arbitrary numbers

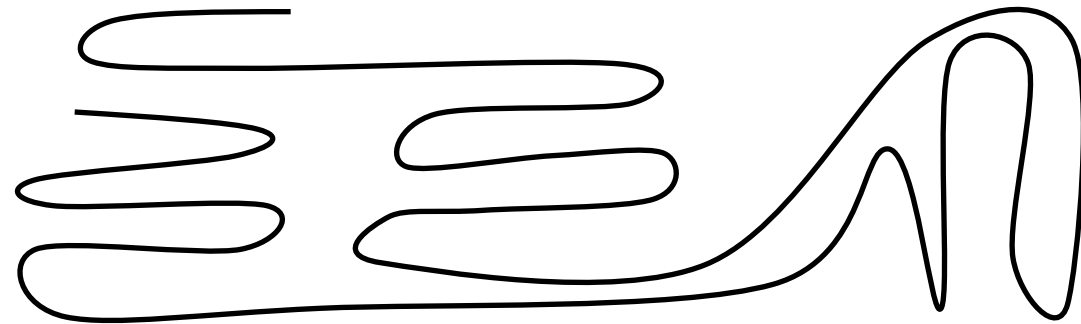
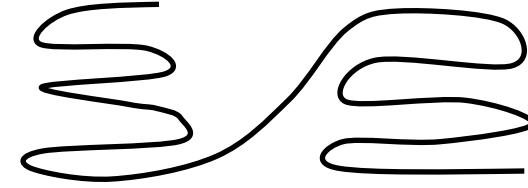


# Why are methods so complicated ?

If we cut protein chain once

- methods are easy - use density criterion

Cut protein twice ? more ? remember  $(N_{res})^{N_{cut}}$

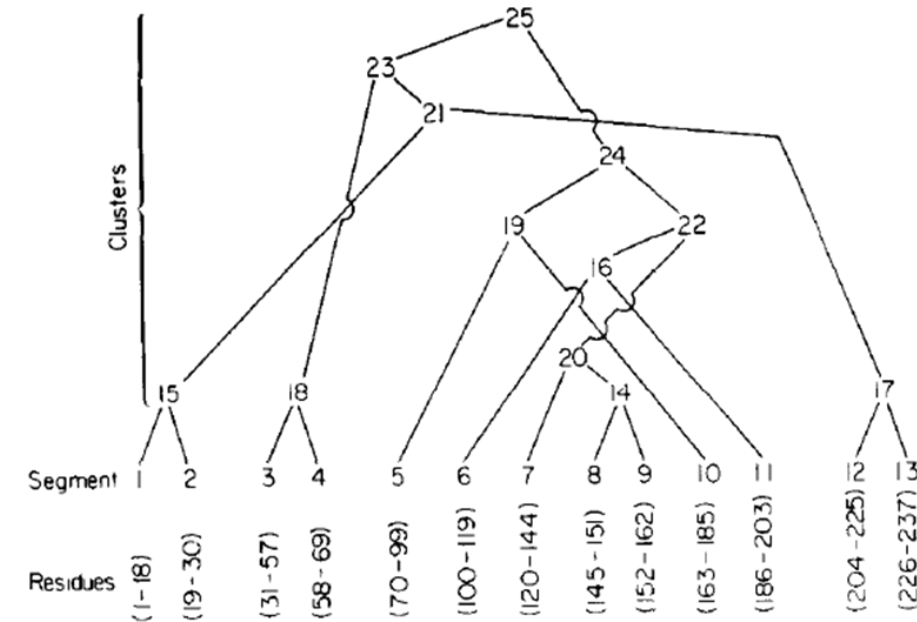


How many domains ?

- Crippen / clustering method - whatever you want
- Network flow - repeatedly split and eventually stop
- Taylor / Ising - may converge to  $> 2$  domains

# Crippen / hierarchical clustering

At what level of hierarchy do I cut tree ?



## Network flow

- what constitutes a contact ? (any 2 atom  $< 4 \text{ \AA}$  ?)
- give pipes (edges) more weight for different kinds of contacts
- are solutions unique ?
  - probably in practice
- when do we stop splitting domains ?

# Taylor / Ising spin method

- what constitutes a contact ? how many Å ?
- type of averaging to get  $m_{av}$  ?
- when does one converge ?

## Elegance

Do methods work as described ? not really

All authors report problems - example

- Taylor finds different results for  $\alpha$ -helical and  $\beta$ -sheet regions
- simple explanation ? distances within / between secondary structure are very different

## **Do methods work ?**

With many fixes and tuning - yes

- distance criteria, thresholds

## **Do methods agree ?**

Only ask question if you agree to think in terms of structure

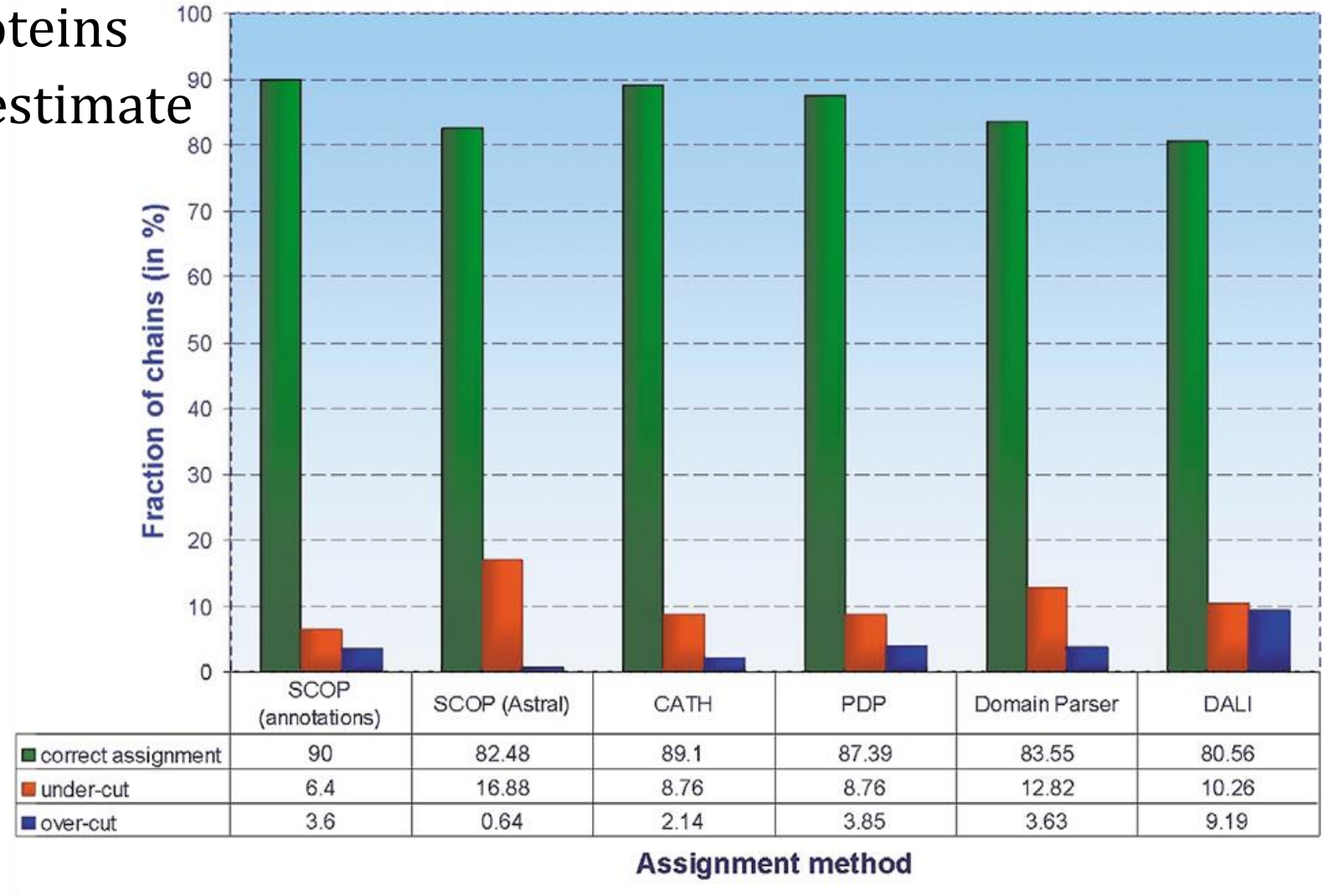
- answer will be different in terms of evolution or sequences

Criteria

- how many domains inside a protein ?
- where are the domain borders ?

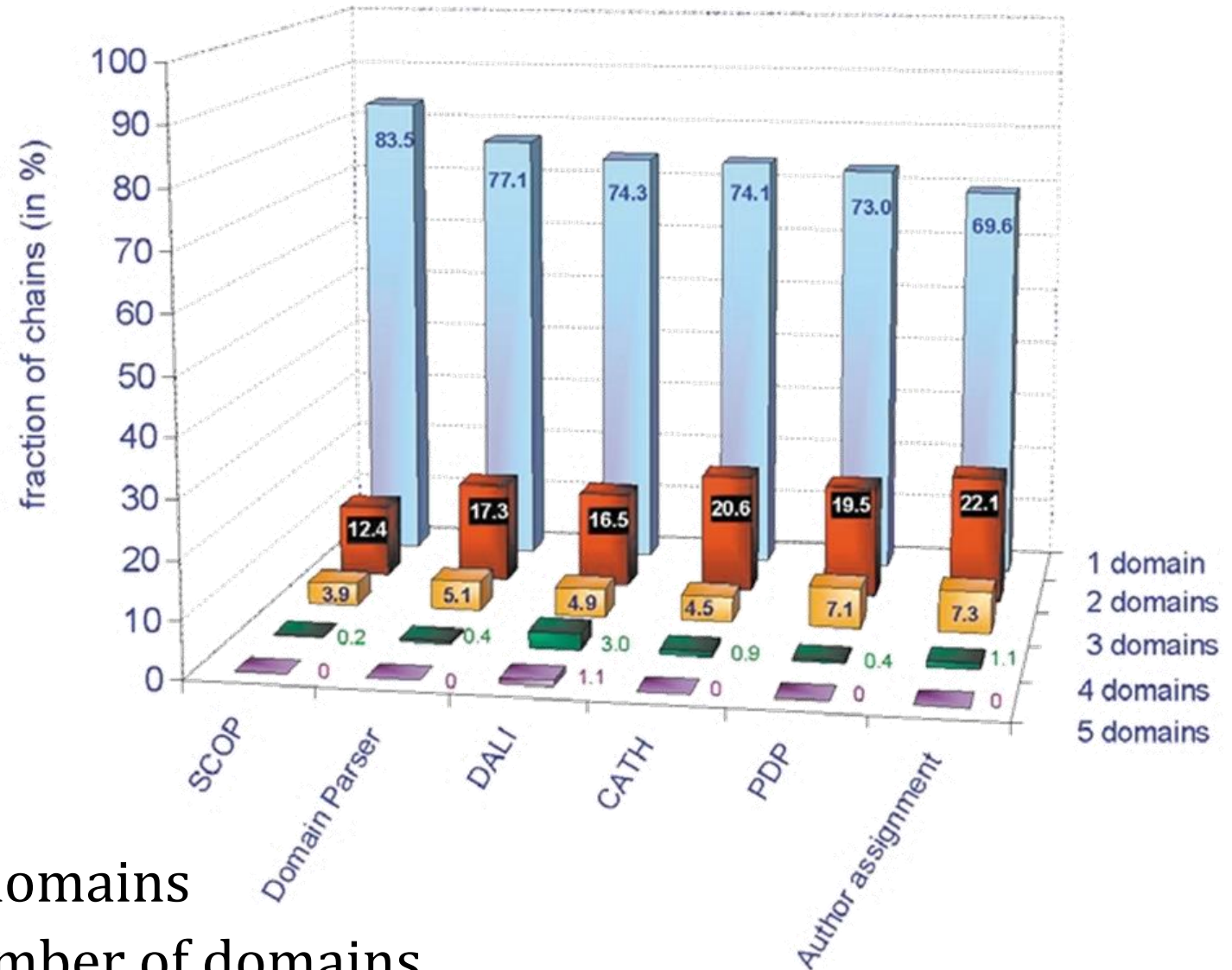
# Number of domains

- test set of few hundred proteins
- compare against author's estimate
- 80-90 % agreement



# How many domains per protein ?

Same set of 467 proteins



- authors split into several domains
- "SCOP" prefers smaller number of domains

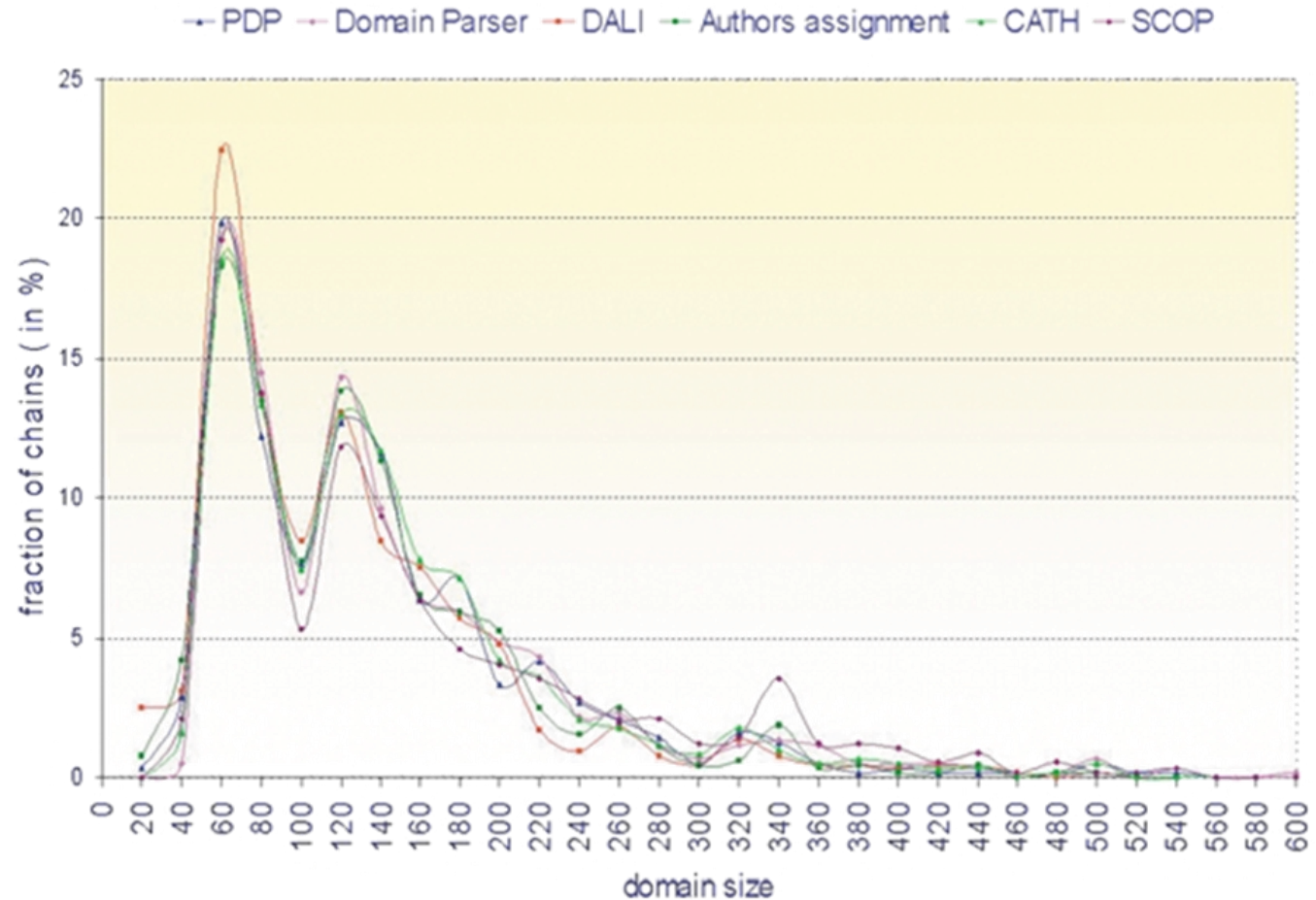
# Agreement ?

Lots of room for differences

## Some statistics

How big is a protein domain ?

Peaks near 60 and 130 residues



# How complicated are domains ?

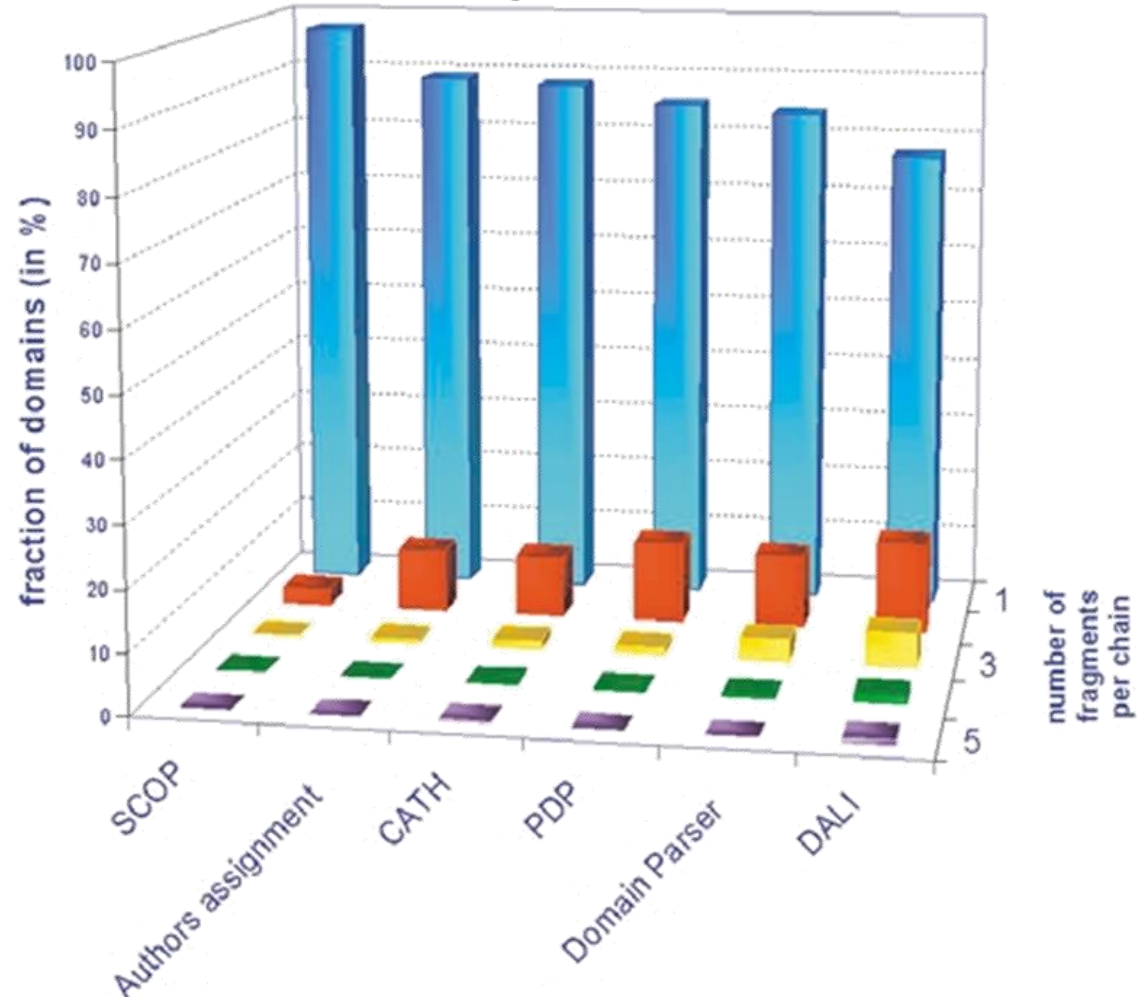
Justification for complicated domain recognition

- single cuts, double cuts in chains are not enough

What percentage of domains are built from

- 1 chain ?
- 2 chains ? ...

In "DALI", 23 % of domains are not continuous (multiple crosses of chain)

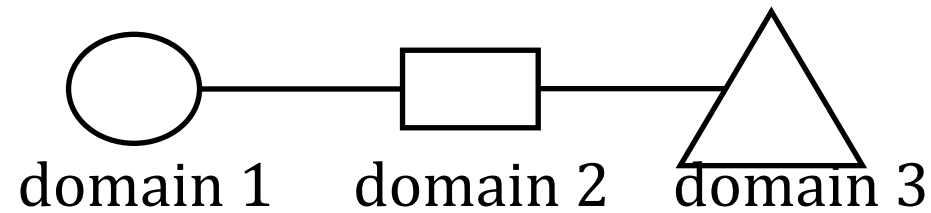




# Evolutionary picture

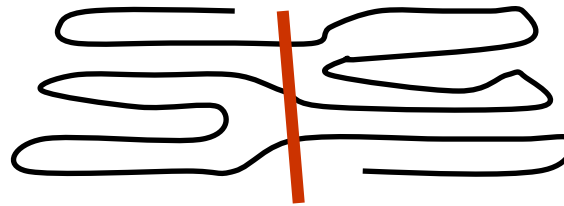
Original claim

- domains are units that move as a module in evolution



If we see multiple cuts 10-20 % of time

- picture is much less clear



# Summary

## Domain definitions

- functional, structural, sequence based

## Finding domains

- relies on contacts, density
- method must be able to handle multiple crossings of chain

## We considered

- clustering / hierarchical
- network flow
- Taylor / Ising spin-inspired
  
- Methods do not agree with each other
- Some trends in size and number of domains
- Real proteins are not as simple as evolutionary picture